Quiz 6
Real Analysis ICTP 2025

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1. Is it true that for $a, b \geq 0$ and $f, g \geq 0$ we have
$$\|af + bg\|_{L^p(\mathbb{R}^d)} = a\|f\|_{L^p(\mathbb{R}^d)} + b\|g\|_{L^p(\mathbb{R}^d)}?$$

2. Is it true that for F, E disjoint we have
$$\|f\|_{L^p(E\cup F)} = \|f\|_{L^p(E)} + \|f\|_{L^p(F)}?$$

3. Is it true for $0 \leq f \leq g$ that $\|f\|_{L^p(\mathbb{R}^d)} \leq \|g\|_{L^p(\mathbb{R}^d)}$?

4. Does $\|f\|_{L^p(\mathbb{R}^d)} = 0$ imply $f = 0$ everywhere?

5. What is another way to write $\|f\|_{L^\infty(\mathbb{R}^d)}$ if $f : \mathbb{R}^d \to \mathbb{R}$ is continuous?

6. If $E \subset \mathbb{R}^2$ is $\mathcal{L}^2$-measurable, is it true that for each $x \in \mathbb{R}$ the set $E_x = \{y \in \mathbb{R} : (x, y) \in E\}$ is $\mathcal{L}^1$-measurable?

7. If $f : \mathbb{R}^2 \to [-\infty, \infty]$ is $\mathcal{L}^2$-integrable, is it true that for every $x \in \mathbb{R}$ the function $y \mapsto f(x, y)$ is $\mathcal{L}^1$-integrable?