Quiz 3
Real Analysis ICTP 2025

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1. Is Lebesgue measure a Radon measure? Is Lebesgue outer measure a Radon outer measure?
2. Do the annuli $A_n = \overline{B(x,n)} \setminus B(x,n-1)$ form a decomposition, i.e. is it true that $\Omega = A_1 \cup A_2 \cup \dots$ and for any $n \neq k$ we have $A_n \cap A_k = \emptyset$?
3. Is it true, that for any $E \subset \Omega \ni x$ and any Radon measure μ that $\mu(E \setminus B(x,n)) \to 0$ as $n \to \infty$?
4. Let μ be the counting measure on $\mathbb{R}$, i.e. $\mu(E)$ equals the number of elements in E if E is finite, and $\mu(E) = \infty$ otherwise. Is μ a Radon measure? Is μ a Radon outer measure?
5. Does Proposition 1.2.18 hold for the counting measure?
6. Is it true that for any measurable $A \subset \mathbb{R}^d$ with $\mathcal{L}(A) < \infty$ and $\varepsilon > 0$ exists a compact set $K \subset A$ with $\mathcal{L}(A \setminus K) < \varepsilon$? What, if $\mathcal{L}(A) = \infty$?
7. Let $E \subset \Omega$ be measurable. Is the characteristic function given by

$$1_E(x) = \begin{cases} 1 & x \in E \\ 0 & x \notin E \end{cases}$$

   measurable?
8. Are polynomials Lebesgue measurable functions?