

Quiz 2

Real Analysis ICTP 2025

Julian Weigt

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1. Does Lebesgue outer measure assign a number to every subset $E \subset \mathbb{R}^d$? Does this number always represent a reasonable notion of volume of the set?
2. Let $(\Omega, \mathcal{M}, \mu)$ be a measure space. Is it true for all $A, B \in \mathcal{M}$ with $A \cap B = \emptyset$ that $\mu_*(A \cup B) = \mu_*(A) + \mu_*(B)$?
3. What is the relation between outer measures and measures?
4. Does $d(x, y) = \sqrt{|x - y|}$ define a metric on $\mathbb{R}$?
5. Let μ_* be a metric outer measure. Is it true for all Borel sets B that $\mu_*(B) < \infty$?
6. Let Q, P be dyadic cubes. What are the options of sets that $P \cap Q$ can be?
7. Does for any Borel set B exist an open set $U \supset B$ with $\mathcal{L}(U) = \mathcal{L}(B)$?