# Quiz 1
# Real Analysis ICTP 2025

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1. For $\mathcal{Q} = \{Q_n : n \in \mathbb{N}\}$ is
$$\bigcup \mathcal{Q} = \bigcup_{n=1}^{\infty} Q_n?$$
2. For $x, y \in \mathbb{R}^d$ and $i \in \{1, ..., d\}$ is it true, that $x_i + y_i = (x + y)_i$?
3. Is it true for all $x \in \mathbb{R}$ that $(x + \infty) - \infty = x + (\infty - \infty)$?
4. In $\mathbb{R}$, what is the value of $\mathcal{L}_*(\{2\})$?
5. Is it true for any set of cubes $Q_1, ..., Q_n \subset \mathbb{R}^d$ that
$$\mathcal{L}_*(Q_1 \cup ... \cup Q_n) = \mathcal{L}_*(Q_1) + ...\mathcal{L}_*(Q_n)?$$
6. Let $\mu_*$ be an outer measure on $\Omega$. Is it true for all sets $A, B \subset \Omega$ that $\mu_*(A \cup B) \leq \mu_*(A) + \mu_*(B)$?
7. Let $\mu_*$ be an outer measure on $\Omega$. Is it true for all sets $A, B \subset \Omega$ with $A \cap B = \emptyset$ that $\mu_*(A \cup B) = \mu_*(A) + \mu_*(B)$?
8. Let $\mu_*$ be an outer measure on $\Omega$. Is it true for all measurable sets $A, B \subset \Omega$ with $A \cap B = \emptyset$ that $\mu_*(A \cup B) = \mu_*(A) + \mu_*(B)$?
9. Does Lebesgue outer measure assign a number to every subset $E \subset \mathbb{R}^d$? Does this number always represent a reasonable notion of volume of the set?